Trigonometry Pre-Calculus Honors Course # 4532 – 4533
Recommended Summer Review Problems

Honors/AP/IB level math courses at Desert Mountain are for students who are enthusiastic learners of mathematics and whose work ethic is of the highest standard. These students are expected to arrive "ready to go" on the first day of school.

The attached packet is designed to help you ***review*** concepts with which you should already be familiar. It is recommended that you complete some of the problems from the packet at the beginning of the summer when the concepts are still fresh, and then complete the remainder of the problems near the beginning of the school year. If you do not complete the problems in the packet, your grade will not be affected directly, however, the material in the packet has been taught in your previous math classes and will be assumed to be fully understood by you. I, the teacher, strongly advise you to work the problems this summer.

The problems will be collected and reviewed after the first week of school once we have gone over any questions you have. If you are new to the Scottsdale Unified School District and did not receive notice of this assignment until registration, these review problems will be checked at the end of August.

Some suggestions for the presentation and completion of mathematics assignments at DMHS are listed below. If you adhere the these guidelines with your summer work, you will be ready to meet the expectations of your mathematics teacher during the school year.

- Use notebook paper or plain white paper
- All working should be neat and legible
- Neatly staple your work or place completed work in a small binder or folder with brads
- Use pencil, erase completely when needed
- Work the problems in order and clearly indicate section and problem numbers
- Begin new sections on a new piece of paper
- Copy the problems and show all of your work
- The odd numbered problems are sufficient for summer practice

1.3 EXERCISES

1–34 ■ Perform the indicated operations and simplify.

1. $(3x^2 + x + 1) + (2x^2 - 3x - 5)$
2. $(3x^2 + x + 1) - (2x^2 - 3x - 5)$
3. $(x^3 + 6x^2 - 4x + 7) - (3x^2 + 2x - 4)$
4. $3(x - 1) + 4(x + 2)$
5. $8(2x + 5) - 7(x - 9)$
6. $4(x^2 - 3x + 5) - 3(x^2 - 2x + 1)$
7. $2(2 - 5t) + t^2(t - 1) - (t^4 - 1)$
8. $5(3t - 4) - (t^2 + 2) - 2t(t - 3)$
9. $\sqrt{x}\,(x - \sqrt{x})$
10. $x^{3/2}\left(\sqrt{x} - 1/\sqrt{x}\right)$
11. $y^{1/3}\,(y^2 - 1)$
12. $y^{1/4}\,(y^{1/2} + 2y^{3/4})$
13. $(3t - 2)(7t - 5)$
14. $(4x - 1)(3x + 7)$
15. $(x + 2y)(3x - y)$
16. $(4x - 3y)(2x + 5y)$
17. $(1 - 2y)^2$
18. $(3x + 4)^2$
19. $(2x^2 + 3y^2)^2$
20. $\left(c + \dfrac{1}{c}\right)^2$
21. $(2x - 5)(x^2 - x + 1)$
22. $(1 + 2x)(x^2 - 3x + 1)$
23. $(x^2 - a^2)(x^2 + a^2)$
24. $(x^{1/2} + y^{1/2})(x^{1/2} - y^{1/2})$
25. $\left(\sqrt{a} - \dfrac{1}{b}\right)\left(\sqrt{a} + \dfrac{1}{b}\right)$
26. $\left(\sqrt{h^2 + 1} + 1\right)\left(\sqrt{h^2 + 1} - 1\right)$
27. $(1 + a^3)^3$
28. $(1 - 2y)^3$
29. $(x^2 + x - 2)(x^3 - x + 1)$
30. $(1 + x + x^2)(1 - x + x^2)$
31. $(1 + x^{4/3})(1 - x^{2/3})$
32. $(1 - b)^2(1 + b)^2$
33. $(3x^2y + 7xy^2)(x^2y^3 - 2y^2)$
34. $(x^4y - y^5)(x^2 + xy + y^2)$

35–88 ■ Factor the expression completely.

35. $12x^3 + 18x$
36. $30x^3 + 15x^4$
37. $6y^4 - 15y^3$
38. $5ab - 8abc$
39. $x^2 - 2x - 8$
40. $x^2 - 14x + 48$
41. $y^2 - 8y + 15$
42. $z^2 + 6z - 16$

43. $2x^2 + 5x + 3$

44. $2x^2 + 7x - 4$

45. $9x^2 - 36x - 45$

46. $8x^2 + 10x + 3$

47. $6x^2 - 5x - 6$

48. $6 + 5t - 6t^2$

49. $4t^2 - 12t + 9$

50. $4x^2 + 4xy + y^2$

51. $r^2 - 6rs + 9s^2$

52. $25s^2 - 10st + t^2$

53. $x^2 - 36$

54. $4x^2 - 25$

55. $49 - 4y^2$

56. $4t^2 - 9s^2$

57. $(a + b)^2 - (a - b)^2$

58. $\left(1 + \frac{1}{x}\right)^2 - \left(1 - \frac{1}{x}\right)^2$

59. $x^2(x^2 - 1) - 9(x^2 - 1)$

60. $(a^2 - 1)b^2 - 4(a^2 - 1)$

61. $t^3 + 1$

62. $x^3 - 27$

63. $8x^3 - 125$

64. $x^6 + 64$

65. $x^6 - 8y^3$

66. $27a^3 + b^6$

67. $x^3 + 2x^2 + x$

68. $3x^3 - 27x$

69. $x^4 + 2x^3 - 3x^2$

70. $x^3 + 3x^2 - x - 3$

71. $y^3 - 3y^2 - 4y + 12$

72. $y^3 - y^2 + y - 1$

73. $2x^3 + 4x^2 + x + 2$

74. $3x^3 + 5x^2 - 6x - 10$

75. $(x - 1)(x + 2)^2 - (x - 1)^2(x + 2)$

76. $(x + 1)^3x - 2(x + 1)^2x^2 + x^3(x + 1)$

77. $y^4(y + 2)^3 + y^5(y + 2)^4$

78. $n(x - y) + (n - 1)(y - x)$

79. $x^{5/2} - x^{1/2}$

80. $3x^{-1/2} + 4x^{1/2} + x^{3/2}$

81. $x^{-3/2} + 2x^{-1/2} + x^{1/2}$

82. $(x - 1)^{7/2} - (x - 1)^{3/2}$

83. $(x^2 + 1)^{1/2} + 2(x^2 + 1)^{-1/2}$

84. $x^{-1/2}(x + 1)^{1/2} + x^{1/2}(x + 1)^{-1/2}$

85. $(a^2 + 1)^2 - 7(a^2 + 1) + 10$

86. $(a^2 + 2a)^2 - 2(a^2 + 2a) - 3$

87–90 ■ Factor the expression completely. (This type of expression arises in calculus when using the "product rule.")

87. $3x^2(4x - 12)^2 + x^3(2)(4x - 12)(4)$

88. $5(x^2 + 4)^4(2x)(x - 2)^4 + (x^2 + 4)^5(4)(x - 2)^3$

89. $3(2x - 1)^2(2)(x + 3)^{1/2} + (2x - 1)^3\left(\frac{1}{2}\right)(x + 3)^{-1/2}$

90. $\frac{1}{3}(x + 6)^{-2/3}(2x - 3)^2 + (x + 6)^{1/3}(2)(2x - 3)(2)$

1–6 ■ Write each radical expression using exponents, and each exponential expression using radicals.

1. $\frac{1}{\sqrt{17}}$ **2.** $\sqrt[4]{7^3}$ **3.** $4^{2/3}$

4. $\sqrt[3]{b^5}$ **5.** $a^{3/5}$ **6.** $w^{-3/2}$

7–14 ■ Evaluate each number.

7. (a) $(-2)^4$ (b) -2^4 (c) $(-2)^0$

8. (a) $\left(\frac{1}{2}\right)^4 4^{-2}$ (b) $\left(\frac{1}{4}\right)^{-2}$ (c) $\left(\frac{1}{4}\right)^0 2^{-1}$

9. (a) $2^4 5^{-2}$ (b) $\frac{10^7}{10^4}$ (c) $(2^3 \cdot 2^2)^2$

10. (a) $\sqrt{64}$ (b) $\sqrt[3]{-64}$ (c) $\sqrt[5]{-32}$

11. (a) $\sqrt{\frac{4}{9}}$ (b) $\sqrt[4]{256}$ (c) $\sqrt[6]{\frac{1}{64}}$

12. (a) $\sqrt{7}\sqrt{28}$ (b) $\frac{\sqrt{48}}{\sqrt{3}}$ (c) $\sqrt[4]{24}\sqrt[4]{54}$

13. (a) $\left(\frac{4}{9}\right)^{-1/2}$ (b) $(-32)^{2/5}$ (c) $(-125)^{-1/3}$

14. (a) $1024^{-0.1}$ (b) $\left(-\frac{27}{8}\right)^{2/3}$ (c) $\left(\frac{25}{64}\right)^{3/2}$

15–18 ■ Evaluate the expression using $x = 3$, $y = 4$, and $z = -1$.

15. $\sqrt{x^2 + y^2}$ **16.** $\sqrt[4]{x^3 + 14y + 2z}$

17. $(9x)^{2/3} + (2y)^{2/3} + z^{2/3}$ **18.** $(xy)^{2z}$

19–22 ■ Simplify the expression.

19. $\sqrt[3]{108} - \sqrt[3]{32}$ **20.** $\sqrt{8} + \sqrt{50}$

21. $\sqrt{245} - \sqrt{125}$ **22.** $\sqrt[3]{54} - \sqrt[3]{16}$

23–40 ■ Simplify the expression and eliminate any negative exponent(s).

23. $a^9 a^{-5}$ **24.** $(3y^2)(4y^5)$

25. $(12x^2y^4)\left(\frac{1}{2}x^5y\right)$ **26.** $(6y)^3$

27. $\frac{x^9(2x)^4}{x^3}$ **28.** $\frac{a^{-3}b^4}{a^{-5}b^5}$

29. $b^4\left(\frac{1}{3}b^2\right)(12b^{-8})$ **30.** $(2s^3t^{-1})\left(\frac{1}{4}s^6\right)(16t^4)$

31. $(rs)^3(2s)^{-2}(4r)^4$ **32.** $(2u^2v^3)^3(3u^3v)^{-2}$

33. $\frac{(6y^3)^4}{2y^5}$ **34.** $\frac{(2x^3)^2(3x^4)}{(x^3)^4}$

35. $\frac{(x^2y^3)^4(xy^4)^{-3}}{x^2y}$ **36.** $\left(\frac{c^4d^3}{cd^2}\right)\left(\frac{d^2}{c^3}\right)^3$

37. $\frac{(xy^2z^3)^4}{(x^3y^2z)^3}$ **38.** $\left(\frac{xy^{-2}z^{-3}}{x^2y^3z^{-4}}\right)^{-3}$

39. $\left(\frac{q^{-1}rs^{-2}}{r^{-5}sq^{-8}}\right)^{-1}$ **40.** $(3ab^2c)\left(\frac{2a^2b}{c^3}\right)^{-2}$

41–56 ■ Simplify the expression and eliminate any negative exponent(s). Assume that all letters denote positive numbers.

41. $x^{2/3}x^{1/5}$ **42.** $(-2a^{3/4})(5a^{3/2})$

43. $(4b)^{1/2}(8b^{2/5})$ **44.** $(8x^6)^{-2/3}$

45. $(c^2d^3)^{-1/3}$ **46.** $(4x^6y^8)^{3/2}$

47. $(y^{3/4})^{2/3}$ **48.** $(a^{2/5})^{-3/4}$

49. $(2x^4y^{-4/5})^3(8y^2)^{2/3}$ **50.** $(x^{-5}y^3z^{10})^{-3/5}$

51. $\left(\frac{x^6y}{y^4}\right)^{5/2}$ **52.** $\left(\frac{-2x^{1/3}}{y^{1/2}z^{1/6}}\right)^4$

53. $\left(\frac{3a^{-2}}{4b^{-1/3}}\right)^{-1}$ **54.** $\frac{(y^{10}z^{-5})^{1/5}}{(y^{-2}z^3)^{1/3}}$

55. $\frac{(9st)^{3/2}}{(27s^3t^{-4})^{2/3}}$ **56.** $\left(\frac{a^2b^{-3}}{x^{-1}y^2}\right)^3\left(\frac{x^{-2}b^{-1}}{a^{3/2}y^{1/3}}\right)$

57–64 ■ Simplify the expression. Assume the letters denote any real numbers.

57. $\sqrt[4]{x^4}$ **58.** $\sqrt[3]{x^3y^6}$

59. $\sqrt[3]{x^3y}$ **60.** $\sqrt{x^4y^4}$

61. $\sqrt[5]{a^6b^7}$ **62.** $\sqrt[3]{a^2b}\sqrt[3]{a^4b}$

63. $\sqrt[3]{\sqrt{64x^6}}$ **64.** $\sqrt[4]{x^4y^2z^2}$

65–68 ■ Rationalize the denominator.

65. (a) $\frac{1}{\sqrt{6}}$ (b) $\sqrt{\frac{x}{3y}}$ (c) $\sqrt{\frac{3}{20}}$

66. (a) $\sqrt{\frac{x^5}{2}}$ (b) $\sqrt{\frac{2}{3}}$ (c) $\sqrt{\frac{1}{2x^3y^5}}$

67. (a) $\frac{1}{\sqrt[3]{x}}$ (b) $\frac{1}{\sqrt[5]{x^2}}$ (c) $\frac{1}{\sqrt[7]{x^3}}$

68. (a) $\frac{1}{\sqrt[3]{x^2}}$ (b) $\frac{1}{\sqrt[4]{x^3}}$ (c) $\frac{1}{\sqrt[3]{x^4}}$

A

1. **a.** If $f(x) = \frac{3}{4}x - \frac{1}{2}$, find $f(2)$ and $f(-2)$. **b.** Find the zero of f.
2. **a.** If $C(n) = 20 - \frac{5}{8}n$, find $C(0)$ and $C(16)$. **b.** Find the zero of C.
3. Let $f(x) = 3x - 7$. Decide whether $f(2) + f(6) = f(8)$.
4. Let $h(t) = \frac{9 - 4t}{2}$. Decide whether $h(4.5) - h(3.5) = h(1)$.
5. Consider the constant functions $g(x) = 2$ and $h(x) = -1$.
   a. Graph the two functions. What are the slopes of their graphs?
   b. Write the functions g and h in the form $f(x) = mx + k$.
6. Consider the constant function $P(x) = -0.5$.
   a. Find $P(1269.35)$.
   b. Does the function P have any zeros? Explain.
7. **a.** What is the slope of the graph of $f(x) = 1.5x - 2$?
   b. What is the zero of the function?
   c. What are the intercepts of the graph?
8. **a.** What is the slope of the graph of $C(t) = 80t + 5.2$?
   b. Where does the graph of the function C intersect the vertical axis?
9. The graph of the linear function f has slope -2 and intersects the n-axis at $n = 6$. Find an equation for $f(n)$.
10. The zero of the linear function S is 3. The graph of this function intersects the vertical axis at $S(x) = -2$. Find an equation for $S(x)$.

1–52 ■ Simplify the expression.

1. $\frac{x - 2}{x^2 - 4}$
2. $\frac{x^2 - x - 2}{x^2 - 1}$
3. $\frac{x^2 + 6x + 8}{x^2 + 5x + 4}$
4. $\frac{x^2 - x - 12}{x^2 + 5x + 6}$
5. $\frac{y^2 + y}{y^2 - 1}$
6. $\frac{y^2 - 3y - 18}{2y^2 + 5y + 3}$
7. $\frac{2x^3 - x^2 - 6x}{2x^2 - 7x + 6}$
8. $\frac{1 - x^2}{x^3 - 1}$
9. $\frac{t - 3}{t^2 + 9} \cdot \frac{t + 3}{t^2 - 9}$
10. $\frac{x^2 - x - 6}{x^2 + 2x} \cdot \frac{x^3 + x^2}{x^2 - 2x - 3}$
11. $\frac{x^2 + 7x + 12}{x^2 + 3x + 2} \cdot \frac{x^2 + 5x + 6}{x^2 + 6x + 9}$

12. $\dfrac{x^2 + 2xy + y^2}{x^2 - y^2} \cdot \dfrac{2x^2 - xy - y^2}{x^2 - xy - 2y^2}$

13. $\dfrac{2x^2 + 3x + 1}{x^2 + 2x - 15} \div \dfrac{x^2 + 6x + 5}{2x^2 - 7x + 3}$

14. $\dfrac{4y^2 - 9}{2y^2 + 9y - 18} \div \dfrac{2y^2 + y - 3}{y^2 + 5y - 6}$

15. $\dfrac{\dfrac{x^3}{x + 1}}{\dfrac{x}{x^2 + 2x + 1}}$

16. $\dfrac{\dfrac{2x^2 - 3x - 2}{x^2 - 1}}{\dfrac{2x^2 + 5x + 2}{x^2 + x - 2}}$

17. $\dfrac{x/y}{z}$

18. $\dfrac{x}{y/z}$

19. $\dfrac{1}{x + 5} + \dfrac{2}{x - 3}$

20. $\dfrac{1}{x + 1} + \dfrac{1}{x - 1}$

21. $\dfrac{1}{x + 1} - \dfrac{1}{x + 2}$

22. $\dfrac{x}{x - 4} - \dfrac{3}{x + 6}$

23. $\dfrac{x}{(x + 1)^2} + \dfrac{2}{x + 1}$

24. $\dfrac{5}{2x - 3} - \dfrac{3}{(2x - 3)^2}$

25. $u + 1 + \dfrac{u}{u + 1}$

26. $\dfrac{2}{a^2} - \dfrac{3}{ab} + \dfrac{4}{b^2}$

27. $\dfrac{1}{x^2} + \dfrac{1}{x^2 + x}$

28. $\dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3}$

29. $\dfrac{2}{x + 3} - \dfrac{1}{x^2 + 7x + 12}$

30. $\dfrac{x}{x^2 - 4} + \dfrac{1}{x - 2}$

31. $\dfrac{1}{x + 3} + \dfrac{1}{x^2 - 9}$

32. $\dfrac{x}{x^2 + x - 2} - \dfrac{2}{x^2 - 5x + 4}$

33. $\dfrac{2}{x} + \dfrac{3}{x - 1} - \dfrac{4}{x^2 - x}$

34. $\dfrac{x}{x^2 - x - 6} - \dfrac{1}{x + 2} - \dfrac{2}{x - 3}$

35. $\dfrac{1}{x^2 + 3x + 2} - \dfrac{1}{x^2 - 2x - 3}$

36. $\dfrac{1}{x + 1} - \dfrac{2}{(x + 1)^2} + \dfrac{3}{x^2 - 1}$

37. $\dfrac{\dfrac{x}{y} - \dfrac{y}{x}}{\dfrac{1}{x^2} - \dfrac{1}{y^2}}$

38. $x - \dfrac{y}{\dfrac{x}{y} + \dfrac{y}{x}}$

39. $\dfrac{1 + \dfrac{1}{c - 1}}{1 - \dfrac{1}{c - 1}}$

40. $1 + \dfrac{1}{1 + \dfrac{1}{1 + x}}$

41. $\dfrac{\dfrac{5}{x - 1} - \dfrac{2}{x + 1}}{\dfrac{x}{x - 1} + \dfrac{1}{x + 1}}$

42. $\dfrac{\dfrac{a - b}{a} - \dfrac{a + b}{b}}{\dfrac{a - b}{b} + \dfrac{a + b}{a}}$

43. $\dfrac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}}$

44. $\dfrac{x^{-1} + y^{-1}}{(x + y)^{-1}}$

45. $\dfrac{1}{1 + a^n} + \dfrac{1}{1 + a^{-n}}$

46. $\dfrac{\left(a + \dfrac{1}{b}\right)^m \left(a - \dfrac{1}{b}\right)^n}{\left(b + \dfrac{1}{a}\right)^m \left(b - \dfrac{1}{a}\right)^n}$

47. $\dfrac{\dfrac{1}{a + h} - \dfrac{1}{a}}{h}$

48. $\dfrac{(x + h)^{-3} - x^{-3}}{h}$

49. $\dfrac{\dfrac{1 - (x + h)}{2 - (x + h)} - \dfrac{1 - x}{2 + x}}{h}$

50. $\dfrac{(x + h)^3 - 7(x + h) - (x^3 - 7x)}{h}$

51. $\sqrt{1 + \left(\dfrac{x}{\sqrt{1 - x^2}}\right)^2}$

52. $\sqrt{1 + \left(x^3 - \dfrac{1}{4x^3}\right)^2}$

53–58 ■ Simplify the expression. (This type of expression arises in calculus when using the "quotient rule.")

53. $\dfrac{3(x + 2)^2(x - 3)^2 - (x + 2)^3(2)(x - 3)}{(x - 3)^4}$

54. $\dfrac{2x(x + 6)^4 - x^2(4)(x + 6)^3}{(x + 6)^8}$

55. $\dfrac{2(1 + x)^{1/2} - x(1 + x)^{-1/2}}{x + 1}$

56. $\dfrac{(1 - x^2)^{1/2} + x^2(1 - x^2)^{-1/2}}{1 - x^2}$

57. $\dfrac{3(1 + x)^{1/3} - x(1 + x)^{-2/3}}{(1 + x)^{2/3}}$

Simplify each expression.

A

1. $\sqrt{-4} + \sqrt{-16} + \sqrt{-1}$
2. $\sqrt{-49} - \sqrt{-9} + \sqrt{-36}$
3. $\sqrt{-1}\sqrt{-9}$
4. $\sqrt{-2}\sqrt{-5}$
5. $\dfrac{\sqrt{-12}}{\sqrt{-3}}$
6. $\dfrac{\sqrt{-25}}{\sqrt{-50}}$
7. $(4 - 3i) + (-6 + 8i)$
8. $(7 - 8i) - (6 + 2i)$
9. $4(3 + 5i) - 2(2 - 6i)$
10. $\frac{1}{6}(7 - 2i) + \frac{2}{3}(5 - 5i)$
11. $(6 - i)(6 + i)$
12. $(7 + 3i)(7 - 3i)$
13. $(5 + i\sqrt{5})(5 - i\sqrt{5})$
14. $(\sqrt{3} + 4i\sqrt{2})(\sqrt{3} - 4i\sqrt{2})$
15. $(8 + 3i)(2 - 5i)$
16. $(5 - 2i)(-1 + 3i)$
17. $(4 - 5i)^2$
18. $(4 + 7i)^2$

Write each expression in the form $a + bi$.

19. $\dfrac{1}{2 + 5i}$
20. $\dfrac{1}{4 - 3i}$
21. $\dfrac{5 + i}{5 - i}$
22. $\dfrac{3 - 2i}{3 + 2i}$
23. $\dfrac{3 + i\sqrt{2}}{7 - i\sqrt{2}}$
24. $\dfrac{2 + i\sqrt{5}}{3 - i\sqrt{5}}$

B

25. $\dfrac{5}{i}$
26. $\dfrac{i^2 + 2i^3}{i}$
27. $i + i^2 + i^3 + i^4 + i^5$
28. $i^{46} + i^{47}$
29. i^{-3}
30. i^{-6}
31. i^{-35}
32. $(i^n)^4$, where n is any integer
33. Find real numbers x and y such that $(2x + y) + (3 - 5x)i = 1 - 7i$.
34. Find real numbers x and y such that $(3x - 4y) + (6x + 2y)i = 5i$.
35. Show that the sum of $a + bi$ and its conjugate is a real number.
36. Show that the product of $a + bi$ and its conjugate is a nonnegative real number.
37. **a.** How could you show that 79 is a square root of 6241 without using a calculator?
    b. How could you show that $3 - i$ is a square root of $8 - 6i$?
38. Show that $4 - 3i$ is a square root of $7 - 24i$.
39. Show that $\dfrac{\sqrt{2}}{2}(1 + i)$ is a square root of i.
40. Find the square roots of $3 + 4i$.

Solve by factoring.

A

1. $3x^2 - 4x - 7 = 0$
2. $4x^2 - 8x - 32 = 0$
3. $(2x - 3)(x + 4) = 6$
4. $(3y - 2)(y + 4) = 24$

Solve by completing the square. Give both real and imaginary roots.

5. $x^2 - 10x = 1575$
6. $x^2 - 6x = 391$
7. $2z^2 - 16z - 1768 = 0$
8. $x^2 - 8x - 20 = 0$
9. $x^2 + 6x + 10 = 0$
10. $y^2 + 10y + 35 = 0$

Solve by using the quadratic formula. Give your answers in simplest radical form. Give both real and imaginary roots.

11. $5x^2 + 2x - 1 = 0$
12. $4x^2 - 4x - 17 = 0$
13. $3t^2 = 12t - 15$
14. $5u^2 + 2 = 5u$
15. $\frac{4}{v} = \frac{v - 6}{v - 4}$
16. $\frac{4}{z} = \frac{3z}{z - 3}$

Solve by whichever method seems easiest. Give both real and imaginary roots. Be sure not to lose or gain roots.

17. $8x^2 = 7 - 10x$
18. $4t = 1 + 15t^2$
19. $(3x - 2)^2 = 121$
20. $(4y + 4)^2 = -16$
21. $(4x + 7)(x - 1) = 2(x - 1)$
22. $(2x + 1)(4x - 3) = 3(4x - 3)^2$
23. $2w(4w - 1) = w(1 - 4w)$
24. $3(2x - 3)^2 = 4x(3 - 2x)$
25. $\frac{x + 3}{x - 3} + \frac{x - 3}{x + 3} = \frac{18 - 6x}{x^2 - 9}$
26. $\frac{r}{r - 1} - \frac{r}{r + 1} = \frac{2}{r^2 - 1}$
27. $\frac{t^2 + 1}{t + 2} = \frac{t}{3} + \frac{5}{t + 2}$
28. $\frac{x + 2'}{x^2 - x - 6} = 3 - \frac{4}{x - 3}$
29. $2\sqrt{x} = x - 8$
30. $\sqrt{2x + 5} = x + 1$

A 1. Find the remainder when $x^5 - 2x^3 + x^2 - 4$ is divided by:
a. $x - 1$ **b.** $x + 1$ **c.** $x - 2$ **d.** $x + 2$

2. Find the remainder when $x^3 - 3x^2 + 5$ is divided by:
a. $x - 2$ **b.** $x + 2$ **c.** $x - 3$ **d.** $x + 3$

In Exercises 3–10, find the quotient and the remainder when the first polynomial is divided by the second.

3. $x^3 - 2x^2 + 5x + 1$; $x - 1$
4. $2x^3 + x^2 + 3x + 7$; $x + 2$
5. $x^4 - 2x^3 + 5x + 2$; $x + 1$
6. $2x^4 - 3x^3 + 4x^2 - 5x + 2$; $x - 1$
7. $x^5 + x^3 + x$; $x - 3$
8. $x^2 - 3x^4$; $x + 2$
9. $3x^4 - 2x^3 + 5x^2 + x + 1$; $x^2 + 2x$
10. $x^5 + 3x^2 + 4$; $x^2 + 2x + 1$

11. Determine whether $x - 1$ or $x + 1$ is a factor of $x^{100} - 4x^{99} + 3$.
12. Determine whether $x - 2$ or $x + 2$ is a factor of $x^{20} - 4x^{18} + 3x - 6$.
13. Which of the following are factors of $P(x) = x^3 - 5x^2 + 3x + 9$?
a. $x - 1$ **b.** $x + 3$ **c.** $x - 3$
14. Which of the following are factors of $P(x) = x^4 - 3x^3 + 5x - 2$?
a. $x + 2$ **b.** $x - 2$ **c.** $x + 4$
15. Show that $x - a$ is a factor of $x^n - a^n$ for any positive integer n.
16. Show that $x + a$ is a factor of $x^n + a^n$ for any positive odd integer n.
17. When a polynomial $P(x)$ is divided by $2x + 1$, the quotient is $x^2 - x + 4$ and the remainder is 3. Find $P(x)$.
18. When a polynomial $P(x)$ is divided by $3x - 4$, the quotient is $x^3 + 2x + 2$ and the remainder is -1. Find $P(x)$.

In Exercises 19–24, you are given a polynomial equation and one or more of its roots. Find the remaining roots.

19. $2x^3 - 5x^2 - 4x + 3 = 0$; root: $x = 3$
20. $6x^3 + 11x^2 - 4x - 4 = 0$; root: $x = -2$

B 21. $2x^4 - 9x^3 + 2x^2 + 9x - 4 = 0$; roots: $x = -1$, $x = 1$
22. $4x^4 - 4x^3 - 25x^2 + x + 6 = 0$; roots: $x = -2$, $x = 3$
23. $x^4 + 3x^3 - 3x^2 + 3x - 4 = 0$; roots: $x = -4$, $x = 1$
24. $x^4 - 2x^3 + x^2 - 4 = 0$; roots: $x = -1$, $x = 2$
25. Use the factor theorem to show that $x - a$ is a factor of
$$x^2(a - b) + a^2(b - x) + b^2(x - a).$$
26. Use the factor theorem to show that $x - c$ is a factor of
$$(x - b)^3 + (b - c)^3 + (c - x)^3.$$

In Exercises 1–12, sketch the graph of each equation.

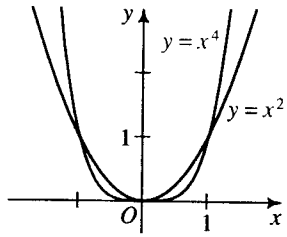
A

1. $y = (x + 1)(x - 2)(x - 4)$
2. $y = -(x + 3)(x + 2)(x - 1)$
3. $y = -x(x + 5)(x + 3)$
4. $y = x(x - 1)(x - 4)$
5. $y = x^2(x + 2)$
6. $y = (x - 1)^3$
7. $y = x(1 - x)(1 + x)(2 + x)$
8. $y = x(x + 2)(x - 2)(x - 1)$
9. $y = (x + 1)^3(x - 2)$
10. $y = -x^2(2 - x)^2$
11. $y = x^2(x + 2)(x - 1)(x + 1)$
12. $y = x^2(1 - x)^2(2 + x)$

In Exercises 13–18, factor each polynomial function and sketch its graph.

13. $f(x) = x^3 - 4x$
14. $f(x) = x^3 - 4x^2 - 5x$
15. $f(x) = x^4 - x^2$
16. $f(x) = x^4 - 2x^3 + x^2$
17. $f(x) = x^4 - 2x^3 + 2x - 1$
    (*Hint*: $x = 1$ is a triple root.)
18. $f(x) = 4x^4 - 24x^3 + 35x^2 + 6x - 9$
    (*Hint*: $x = 3$ is a double root.)

19. The graphs of $y = x^2$ and $y = x^4$ are shown at the right.
    a. Copy these graphs and then add the graph of $y = x^6$. (You may wish to use a computer or graphing calculator to check your graph.)
    b. What three points are common to all three graphs?



20. The graphs of $y = x^3$ and $y = x^5$ are shown at the right.
    a. Copy these graphs and then add the graph of $y = x^7$. (You may wish to use a computer or graphing calculator to check your graph.)
    b. What three points are common to all three graphs?