Course #4554-4555 — Calculus BC AP
Course #5026-5027 — IB Math A&A (HL year 2)

Desert Mountain H. S. Math Department

Summer Work Packet

Honors/AP/IB level math courses at Desert Mountain are for students who are enthusiastic learners of mathematics and whose work ethic is of the highest standard. These students are expected to arrive "ready to go" on the first day of school.

The attached packet is designed to help you ***review*** concepts with which you should already be familiar. It is recommended that you complete some of the problems from the packet at the beginning of the summer when the concepts are still fresh, and then complete the remainder of the problems near the beginning of the school year. If you do not complete the problems in the packet, your grade will not be affected directly, however, the material in the packet has been taught in your previous math classes and will be assumed to be fully understood by you. I, the teacher, strongly advise you to work the problems this summer.

The problems will be collected and reviewed after the first week of school once we have gone over any questions you have. If you are new to the Scottsdale Unified School District and did not receive notice of this assignment until registration, these review problems will be checked at the end of August.

Some suggestions for the presentation and completion of mathematics assignments at DMHS are listed below. If you adhere to these guidelines with your summer work, you will be ready to meet the expectations of your mathematics teacher during the school year.

- Use notebook paper or plain white paper
- All working should be neat and legible
- Neatly staple your work or place completed work in a small binder or folder with brads
- Use pencil, erase completely when needed
- Work the problems in order and clearly indicate section and problem numbers
- Begin new sections on a new piece of paper

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Basic Differentiation Rules

(Memorize all of these, or else!)

1. $\frac{d}{dx}\; cu = cu'$

2. $\frac{d}{dx}\; u \pm v = u' \pm v'$

3. $\frac{d}{dx}\; uv = uv' + vu'$

4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$

5. $\frac{d}{dx}\; c = 0$

6. $\frac{d}{dx}\; u^n = nu^{n-1}u'$

7. $\frac{d}{dx}\; x = 1$

8. $\frac{d}{dx}\; |u| = \frac{u}{|u|}u'$

9. $\frac{d}{dx}\; \ln u = \frac{u'}{u}$

10. $\frac{d}{dx}\; e^u = e^u u'$

11. $\frac{d}{dx}\; \sin u = (\cos u)u'$

12. $\frac{d}{dx}\; \cos u = -(\sin u)u'$

13. $\frac{d}{dx}\; \tan u = (\sec^2 u)u'$

14. $\frac{d}{dx}\; \cot u = -(\csc^2 u)u'$

15. $\frac{d}{dx}\; \sec u = (\sec u \tan u)u'$

16. $\frac{d}{dx}\; \csc u = -(\csc u \cot u)u'$

17. $\frac{d}{dx}\; \arcsin u = \frac{u'}{\sqrt{1-u^2}}$

18. $\frac{d}{dx}\; \arccos u = \frac{-u'}{\sqrt{1-u^2}}$

19. $\frac{d}{dx}\; \arctan u = \frac{u'}{1+u^2}$

20. $\frac{d}{dx}\; arc\cot u = \frac{-u'}{1+u^2}$

21. $\frac{d}{dx}\; arc\sec u = \frac{u'}{|u|\sqrt{u^2-1}}$

22. $\frac{d}{dx}\; arc\csc u = \frac{-u'}{|u|\sqrt{u^2-1}}$

23. $\frac{d}{dx}\left[a^x\right] = a^x \ln a$

24. $\frac{d}{dx}\left[\log_b x\right] = \frac{1}{x \ln b}$

** Be ready for a pop quiz on these as early as the first day of school. Hee hee… =)

Linear Equations

Write the following equations in point-slope form $y = m(x - x_1) + y_1$

1. The line containing the point $(4,-7)$ and having a slope of $\frac{5}{2}$.
2. The line containing the point $(0,-2)$ and perpendicular to $x - 4y = 3$.
3. The line containing the point $(2,9)$ and having a slope of 0.
4. The perpendicular bisector of the segment between $(-5,3)$ and $(12,3)$.

Compositions of Functions

Given $f(x) = 4x - 1$ and $g(x) = x + 6$, find the following compositions.

5. $g(f(x))$
6. $f(g(x))$
7. $f(f(x))$

Basic Factoring

Factor each of the following as completely as possible.

8. $9x^3y - 25xy^3$
9. $x^3 + 7x^2 - 18x$
10. $8y^3 + 24y^2 - 7y - 21$

Function Analysis

Determine the domain and zeros of the each of the following functions Then use those values on a sign chart (number line) to determine intervals where the function is positive and negative.

11. $P(x)=(x+5)(x-8)$

12. $C(x)=\dfrac{-6}{2x-3}$

13. $f(x)=\dfrac{x+1}{x+2}$

14. $T(x)=\dfrac{(x-3)(x+2)^2}{(x-10)^3}$

Mixed Review Problems

15. Find all roots of $P(x)=3x^3+x^2+12x+4$ using factor by grouping.

16. Solve by hand: $\log_4(x)=3$.

17. Express as a single logarithm: $3\log_b\left(\sqrt[3]{x}\right)-2\log_b(x)$.

18. Solve $\sqrt{4y-9}-\sqrt{5y-4}=1$.

19. Solve $\log_3(x+3)+\log_3(x-3)=4$.

20. Simplify $\dfrac{y-\frac{1}{y}}{y+\frac{1}{y}}$.

21. Determine the amplitude and period of $y=3\sin 2x$ and sketch the graph.

22. Simplify $\dfrac{\tan x\cos^2 x+\tan x\sin^2 x}{\sin x}$.

23. Given $\sin\theta=-\dfrac{4}{5}$ and $\pi<\theta<\dfrac{3\pi}{2}$, find $\sin 2\theta, \cos 2\theta$, and $\tan 2\theta$.

24. Solve $2\sin^2 x=1$ on $[0,2\pi]$.

25. Solve $\cos x=\sin 2x$ on $[0,2\pi]$. (hint: use the double-angle identify first and don't cancel)

Trigonometry

You should complete these problems *without* the use of a calculator.

Evaluate.

1. $\sin(120°)$

2. $\cos\left(\frac{5\pi}{4}\right)$

3. $\tan(-60°)$

4. $\sec\left(\frac{\pi}{2}\right)$

5. $\cot(60°)$

6. $\csc\left(\frac{4\pi}{3}\right)$

7. Sketch a complete graph of $y = \cos x$. Label important places (like *x*-intercepts).

8. State the domain of $y = \sin x$ and $y = \csc x$.

9. Given that $\sin\phi = -\frac{5}{13}$ in the 3rd quadrant, find the values of the other 5 trig. functions.

Solve for *x*.

10. $2\sin x - 1 = 0$

11. $\sin x + \sqrt{2} = -\sin x$

Calculus:

These are some of the most important questions, so don't wait until the last minute, and don't decide to leave them blank. That would make me mad. *Make sure you spend time actually thinking about them before you write anything down.* Read them now and consider them as you work through the packet. You probably can't answer them all right now.

1. What does $f'(x)$ tell you about $f(x)$?

2. Is it possible for a function to be continuous, but not differentiable? If so, give an example.

3. Is it possible for a function to be differentiable, but not continuous? If so, give an example.

4. If the limit of a function exists at $x = c$, then must the function must be defined at $x = c$? Explain why or why not.

5. What is the difference between removable and non-removable discontinuities? How is that related to limits?

6. Let $f(x) = e^x$. Find the inverse of f and call it g.

   a) Find and compare $f'(1)$ and $g'(e)$.

   b) Find and compare $f'(2)$ and $g'(e^2)$.

   c) Use your answers to (a) and (b) to discuss the relationship between the slope on a function and the slope on its inverse.

   d) Now go back to section 2.6 and re-attempt problems #1-7 using what you have learned. The other questions in 2.6 can be answered using the derivative rules listed on the second page.

7. What exactly does *velocity* measure? If the *velocity* of an object is negative, what does that mean? If the *velocity* of an object is negative, does that also mean that the object is slowing down? Explain why and/or why not.

8. When it is said that the derivative of $f(x)$ is positive at $x = c$, what does that really mean about $f(x)$?

9. What is the meaning of the following statement? Your explanation should include some discussion of inputs and outputs of the function. $\lim_{x\to 3} f(x) = 4$

Section 1.2

In Exercise 5, complete the table and use the result to estimate the limit.

5. $\lim_{x \to 0} \frac{\sin x}{x}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

In exercises 11-18, graph the equation on your graphing calculator. Then use your graph to find the limit (if it exists). If the limit does not exist, explain why.

11. $\lim_{x \to 3} (4 - x)$

12. $\lim_{x \to 2} \frac{1}{x - 2}$

13. $\lim_{x \to 3} \frac{|x - 3|}{x - 3}$

14. $\lim_{x \to 1} f(x)$; $f(x) = \begin{cases} x^2 + 3, & x \neq 1 \\ 1, & x = 1 \end{cases}$

16. $\lim_{x \to 0} \frac{4}{2 + e^{1/x}}$

17. $\lim_{x \to \pi/2} \tan x$

18. $\lim_{x \to 0} 2\cos\frac{1}{x}$

43. Write a brief description of the meaning of the notation $\lim_{x \to 8} f(x) = 25$.

True or False? In exercises 49 and 51, determine whether the statement is true or false. If it is false, explain why or give an example that shows it's false.

49. If *f* is undefined at $x = c$, then the limit of $f(x)$ as x approaches c does not exist.

51. If $f(c) = L$, then $\lim_{x \to c} f(x) = L$.

Section 1.3

For exercises 5-13, find the limit.

5. $\lim_{x \to 2} x^4$

9. $\lim_{x \to -3} (2x^2 + 4x + 1)$

13. $\lim_{x \to 1} \frac{x - 3}{x^2 + 4}$

Use the information to evaluate the limit.

37. $\lim_{x \to c} f(x) = 2$, $\lim_{x \to c} g(x) = 3$

a) $\lim_{x \to c} 5g(x)$

b) $\lim_{x \to c} f(x) + g(x)$

c) $\lim_{x \to c} f(x)g(x)$

d) $\lim_{x \to c} \frac{f(x)}{g(x)}$

Graph the function to determine the limit visually if it exists. Write a simpler function that agrees with the given function at all but one point. (note: there is a hole at (0,1))

41. a) $\lim_{x \to 0} \frac{-2x^2 + x}{x}$

Find the limit of the function (by hand) if it exists. Write a simpler function that agrees with the given function at all but one point. Use a graphing utility to confirm your results.

45. $\lim_{x \to -1} \frac{x^2 - 1}{x + 1}$

49. $\lim_{x \to -4} \frac{(x+4)\ln(x+6)}{x^2 - 16}$

For exercises 53, 57, 61, find the limit (by hand) if it exists.

53. $\lim_{x \to -3} \frac{x^2 + x - 6}{x^2 - 9}$

57. $\lim_{x \to 4} \frac{\sqrt{x+5} - \sqrt{5}}{x}$

61. $\lim_{\Delta x \to 0} \frac{2(x + \Delta x) - 2x}{\Delta x}$

Determine the limit of the transcendental function (by hand) if it exists.

69. $\lim_{x \to 0} \frac{\sin x}{5x}$

70. $\lim_{x \to 0} \frac{5(1 - \cos x)}{x}$

71. $\lim_{x \to 0} \frac{\sin x(1 - \cos x)}{2x^2}$

72. $\lim_{\theta \to 0} \frac{\cos\theta \tan\theta}{\theta}$

73. $\lim_{x \to 0} \frac{\sin^2 x}{x}$

74. $\lim_{x \to 0} \frac{2\tan^2 x}{x}$

89. Let $f(x) = 2x + 3$. Find $\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

119. $\lim_{x \to 0} \frac{|x|}{x} = 1$

123. $\lim_{x \to 2} f(x) = 3$, Where $f(x) = \begin{cases} 3 & x \le 2 \\ 0 & x > 2 \end{cases}$

Section 1.4

Find the limit if it exists. If the limit does not exist, then explain why.

7. $\lim_{x \to 5^+} \frac{x - 5}{x^2 - 25}$

9. $\lim_{x \to -3^-} \frac{x}{\sqrt{x^2 - 9}}$

11. $\lim_{x \to 0^-} \frac{|x|}{x}$

Use a graphing utility to obtain a graph of each function. Then state the interval(s) on which the function is continuous.

29. $f(x)=\dfrac{1}{x^2-4}$

30. $f(x)=\dfrac{x^2-1}{x+1}$

32. $f(x)=\begin{cases} x & x<1 \\ 2 & x=1 \\ 2x-1 & x>1 \end{cases}$

Discuss the continuity of the function on the closed interval.

35. $f(x)=\begin{cases} 3-x & x\le 0 \\ 3+0.5x & x>0 \end{cases}$; [-1, 4]

Find the *x*-values, if any, at which *f* is not continuous. Which of the discontinuities are removable?

43. $f(x)=\dfrac{x}{x^2+1}$

45. $f(x)=\dfrac{x+2}{x^2-3x-10}$

53. $f(x)=\begin{cases} \tan\dfrac{\pi x}{4} & |x|<1 \\ x & |x|\ge 1 \end{cases}$

Find the constants, *a*, such that the function is continuous on the entire real line.

66. $g(x)=\begin{cases} \dfrac{x^2-a^2}{x-a}, & x\ne a \\ 8, & x=a \end{cases}$

Explain why the function must have a zero in the specified interval.

83. $f(x)=x^2-4x+3, \quad 2,4$

Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

108. If $\lim\limits_{x\to c} f(x)=L$ and $f(c)=L,$ then *f* is continuous at *c*.

Section 1.5

Use the graph to determine whether *f*(*x*) approaches ∞ or -∞ as *x* approaches –2 from the left and from the right.

2. $f(x)=\dfrac{-1}{x+2}$

Determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches –3 from the left and from the right by completing the table. Use a graphing utility to graph the function and confirm your answer.

x	-3.5	-3.1	-3.01	-3.001
$f(x)$				

x	-2.999	-2.99	-2.9	-2.5
$f(x)$				

5. $f(x)=\dfrac{1}{x^2-9}$

Find the vertical asymptotes of the function.

9. $f(x)=\dfrac{1}{x^2}$

11. $h(x)=\dfrac{x^2-2}{x^2-x-2}$

15. $g(t)=\dfrac{t-1}{t^2+1}$

17. $f(x)=\tan 2x$

Determine whether the function has a vertical asymptote or a removable discontinuity at $x = -1$. Graph the function using a graphing utility to confirm your answer.

33. $f(x)=\dfrac{x^2-1}{x+1}$

35. $f(x)=\dfrac{x^2+1}{x+1}$

Find the limit.

47. $\lim\limits_{x\to 0^-}\left(1+\dfrac{1}{x}\right)$

49. $\lim\limits_{x\to 0^+}\dfrac{2}{\sin x}$

67. *Illegal Drugs:* The cost in millions of dollars for a governmental agency to seize x% of an illegal drug is

$$C=\frac{528x}{100-x},\ \ 0\le x<100$$

a) Find the cost of seizing 25% of the drug.
b) Find the cost of seizing 50% of the drug.
c) Find the cost of seizing 75% of the drug.
d) Find the limit of C as $x\to 100^-$ and interpret the meaning.

Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

74. If $p(x)$ is a polynomial, then $f(x)=\frac{p(x)}{x-1}$ has a vertical asymptote at $x = 1$.

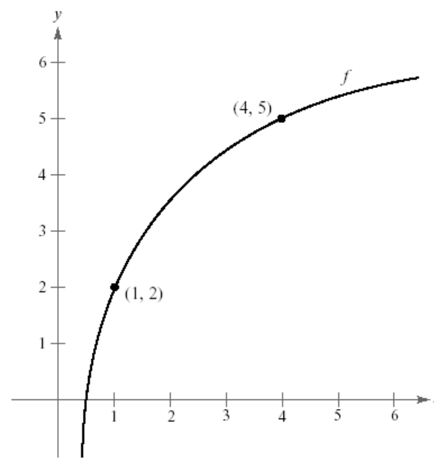
75. A rational function has at least one vertical asymptote.

76. Polynomial functions have no vertical asymptotes.

77. If f has a vertical asymptote at $x = 0$, then f is undefined at $x = 0$.

Section 2.1

For problems 3 and 4, use the graph sown in the figure.



3. Identify or sketch each of the quantities on the figure.
   a) $f(1)$ and $f(4)$
   b) $f(4)$ - $f(1)$
   c) $y=\frac{f(4)-f(1)}{4-1}(x-1)+f(1)$

4. Insert the proper inequality symbol (< or >) between the given quantities.

   a) $\frac{f(4)-f(1)}{4-1}$ ______ $\frac{f(4)-f(3)}{4-3}$

   b) $\frac{f(4)-f(1)}{4-1}$ ______ $f'(1)$

Find the slope of the tangent line to the graph of the function at the specified point.

5. $f(x)=3-2x$, (-1, 5)   10. $h(t)=t^2+3$, (-2, 7)

Find the derivative by the limit process.

11. $f(x)=3$ 13. $f(x)=-5x$ 21. $f(x)=\frac{1}{x-1}$

Find an equation of the line that is tangent to the graph of f and parallel to the given line.

33. *Function:* $f(x)=x^3$ *Line:* $3x-y+1=0$

43. Sketch a graph of a function whose derivative is always negative.

Use the alternative form of the derivative to find the derivative at $x = c$, if it exists.

61. $f(x) = x^2 - 1,\ c = 2$

Use your graphing utility to obtain a graph of each function. Describe the *x*-values at which the function is differentiable.

71. $f(x) = |x+3|$

72. $f(x) = |x^2 - 9|$

73. $f(x) = \dfrac{1}{x+1}$

75. $f(x) = (x-3)^{2/3}$

77. $f(x) = \sqrt{x-1}$

80. $f(x) = \begin{cases} x^2 - 2x & x > 1 \\ x^3 - 3x^2 + 3x & x \le 1 \end{cases}$

Find the derivatives from the left and from the right at $x = 1$ if they exist. Is the function differentiable at $x = 1$?

84. $f(x) = \begin{cases} x & x \le 1 \\ x^2 & x > 1 \end{cases}$

Section 2.2

Find the derivative of the function.

5. $y = x^6$

9. $f(x) = x + 1$

13. $g(x) = x^2 + 4x^3$

17. $f(x) = 6x - 5e^x$

21. $y = x^2 - \dfrac{1}{2}\cos x$

Complete the table.

25.

Original Function	Rewrite	Differentiate	Simplify
$y = \dfrac{5}{2x^2}$	$y = \dfrac{5}{2}x^{-2}$		

29.

Original Function	Rewrite	Differentiate	Simplify
$y = \dfrac{\sqrt{x}}{x}$			

Find the slope of the graph of the function at the indicated point. Use the derivative feature of a graphing utility to confirm your results.

31.	$f(x) = \dfrac{3}{x^2}$	(1, 3)
35.	$f(\theta) = 4\sin\theta - \theta$	(0, 0)

38.

$g(x) = -4e^x$	(1, -4e)

For problems 53 and 55:

a) Find an equation of the tangent line to the graph of *f* at the indicated point.
b) Use a graphing utility to graph the function and its tangent line at the point.
c) Use the derivative feature of a graphing utility to confirm your results.

53. $y = x^4 - x$ at (-1, 2) 55. $g(x) = x + e^x$ at (0, 1)

Determine the points if any at which the graph of the function has a horizontal tangent line.

57. $y = x^4 - 8x^2 + 2$

Find *k* such that the line is tangent to the graph of the function.

63. Function: $f(x) = x^2 - kx$ Line: $y = 4x - 9$

Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

81. If *f'(x) = g'(x),* then *f(x) = g(x).*

82. If *f(x) = g(x) + c*, then *f'(x) = g'(x).*

83. If $y = \pi^2$, then *dy/dx* = 2 π.

Find the average rate of change of the function over the indicated interval. Compare this average rate of change with the instantaneous rates of change at the endpoints of the interval.

87. Function: $f(x) = \frac{-1}{x}$, Interval: 1,2

88. Function: $f(x) = \cos x$, Interval: $\left[0, \frac{\pi}{3}\right]$

Vertical Motion Use the position function $s(t) = -16t^2 + v_0 t + s_0$ for free-falling objects.

91. A silver dollar is dropped from the top of a building that is 1362 feet tall.
a) Determine the position and velocity functions for the coin.
b) Determine the average velocity on the interval [1, 2].
c) Find the instantaneous velocities when t = 1 and t = 2.
d) Find the time required for the coin to reach ground level.
e) Find the velocity of the coin at impact.

Section 2.3 (Product and Quotient Rules)

Use the Product Rule to differentiate the function.

1. $g(x) = (x^2+2)(x^2-3x)$

Use the Quotient Rule to differentiate the function.

5. $f(x) = \dfrac{x}{x^2+1}$

Find $f'(x)$ and $f'(c)$

13. $f(x) = x\cos x \quad c = \dfrac{\pi}{4}$

Complete the table without using the Quotient Rule.

17.

Function	Rewrite	Differentiate	Simplify
$y = \dfrac{x^2+2x}{3}$			

21.

Function	Rewrite	Differentiate	Simplify
$y = \dfrac{4x^{3/2}}{x}$			

35. Find $f'(x)$ if $f(x) = \dfrac{x^2+c^2}{x^2-c^2}$, where c is a constant.

Find the derivative of the transcendental function.

37. $f(t) = t^2 \sin t$

41. $f(x) = -e^x + \tan x$

Evaluate the derivative of the function at the indicated point. Use a graphing utility to verify your result.

60. Function: $f(x) = \tan x \cot x$ Point: (1, 1)

61. Function: $h(t) = \dfrac{\sec t}{t}$ Point: $\left(\pi, -\dfrac{1}{\pi}\right)$

Prove the following differentiation rules (hint: first re-write the given function as a quotient).

81. a) $\frac{d}{dx}\sec x = \sec x \tan x$ b) $\frac{d}{dx}\csc x = -\csc x \cot x$ c) $\frac{d}{dx}\cot x = -\csc^2 x$

Find the second derivative of the function.

83. $f(x) = 4x^{3/2}$ 89. $g(x) = \frac{e^x}{x}$

Find $f'(2)$ for the functions in #97-99 given the following,

$g(2) = 3$ and $g'(2) = -2$

$h(2) = -1$ and $h'(2) = 4$

97. $f(x) = 2g(x) + h(x)$ 98. $f(x) = 4 - h(x)$ 99. $f(x) = \frac{g(x)}{h(x)}$ 100. $f(x) = g(x)h(x)$

Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

111. If $y = f(x)g(x)$, then $\frac{dy}{dt} = f'(x)g'(x)$.

114. If *f(x)* is an *n*th-degree polynomial, then $f^{(n+1)}(x) = 0$

Section 2.4 (Chain Rule)

Find the derivative of the function.

9. $y = (2x-7)^3$ 17. $y = \sqrt[3]{9x^2+4}$ 21. $y = \frac{1}{x-2}$

25. $y = \frac{1}{\sqrt{x+2}}$ 29. $y = x\sqrt{1-x^2}$ 59. $f(\theta) = \frac{1}{4}\sin^2 2\theta$

63. $y = \sin(\cos x)$ 67. $y = e^{\sqrt{x}}$ 71. $y = \ln(e^{x^2})$ 87. $f(x) = \ln\left(\frac{x}{x^2+1}\right)$

Find the derivative of the function.

121. $f(x) = 4^x$ 125. $g(t) = t^2 2^t$ 129. $y = \log_3 x$

133. $y = \log_5 \sqrt{x^2 - 1}$

The relationship between *f* and *g* is given. State the relationship between *f'* and *g'*.

141. $g(x) = f(3x)$

143. Given that $g(5) = -3$, $g'(5) = 6$, $h(5) = 3$, and $h'(5) = -2$, find $f'(5)$ (if possible) for each of the following. If it is not possible, state what additional information is required.

a) $f(x) = g(x)h(x)$ b) $f(x) = g(h(x))$

c) $f(x) = \dfrac{g(x)}{h(x)}$ d) $f(x) = g(x)^{3}$

159. *Depreciation* After *t* years, the value of a car purchased for \$20,000 is $V(t) = 20{,}000\left(\frac{3}{4}\right)^t$

a) Use a graphing utility to graph the function and determine the value of the car 2 years after it was purchased.

b) Find the rate of change of *V* with respect to *t* when *t* = 1 and *t* = 5.

Section 2.5 (Implicit Differentiation)

Find *dy/dx* by implicit differentiation.

1. $x^2 + y^2 = 36$ 7. $xe^y - 10x + 3y = 0$ 16. $\cot y = x - y$ 17. $y = \sin(xy)$

Find *dy/dx* by implicit differentiation and evaluate the derivative at the indicated point.

25. Equation: $xy = 4$ Point: (-4, -1)

Find d^2y/dx^2 in terms of *x* and *y*.

41. $x^2 + y^2 = 36$ 44. $1 - xy = x - y$

Find the points at which the graph of the equation has a vertical or horizontal tangent line.

53. $25x^2 + 16y^2 + 200x - 160y + 400 = 0$

Find *dy/dx* using logarithmic differentiation.

55. $y = x\sqrt{x^2 - 1}$ 61. $y = x^{2/x}$